Similarly, if $\cos\alpha_s$, $\cos\beta_s$ and $\cos\gamma_s$ are the direction cosines of the shock surface with respect to the x, y, and z axes, respectively, then the jump condition across the shock becomes

$$\left\langle\left(1-M_\infty^2\frac{\partial\varphi}{\partial x}\right)S_t\right\rangle+\cos\alpha_s\left\langle 1+(1-M_\infty^2)\frac{\partial\varphi}{\partial x}\right.$$

$$\left.-\frac{\lambda}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2-\frac{1}{2}M_\infty^2\left(\frac{\partial\varphi}{\partial y}\right)^2-\frac{1}{2}M_\infty^2\left(\frac{\partial\varphi}{\partial z}\right)^2\right\rangle$$

$$+\cos\beta_s\left\langle\left(1-M_\infty^2\frac{\partial\varphi}{\partial x}\right)\frac{\partial\varphi}{\partial y}\right\rangle+\cos\gamma_s\left\langle\left(1-M_\infty^2\frac{\partial\varphi}{\partial x}\right)\frac{\partial\varphi}{\partial z}\right\rangle=0 \tag{20}$$

The equations previously derived are for general three-dimensional finite wings.

Equation (16) can be further simplified for wings of aspect ratio of the order of one, or of any aspect ratio but appreciable sweep. Following exactly the analysis given by Hall and Firmin,[6]

$$2KM_\infty^2\frac{\partial^2\varphi}{\partial x\partial t}=[1-M_\infty^2-\lambda\varphi_x]\varphi_{xx}$$

$$+\varphi_{yy}+\varphi_{zz}-2\varphi_y\varphi_{xy}-2\varphi_z\varphi_{xz} \tag{21}$$

For $b\rightarrow\infty$, i.e., for a flow approaching that past a two-dimensional airfoil, $\varphi_{yy}\rightarrow 0$ and $\varphi_y\varphi_{xy}\rightarrow 0$, while $\varphi_z\varphi_{xz}$ $(\sim\epsilon^2/c^2)\ll\varphi_{zz}(\sim\epsilon/c^2)$ and the equation reduces to

$$[1-M_\infty^2-\lambda\varphi_x]\varphi_{xx}+\varphi_{zz}=2KM_\infty^2\varphi_{xt} \tag{22}$$

Equation (21) is the same as was taken by Hall and Firmin[6] except for the value of λ, where it was taken as

$$\lambda=(\gamma+1)M_\infty^2 \tag{23}$$

The Eq. (22) has been studied in Ref. 2. The steady-state equation corresponding to Eq. (16) with the same value of λ has been studied in NLR (Ref. 1), where the cross terms $\varphi_z\varphi_{xz}$ and $\varphi_x\varphi_{zz}$, which are permissible for large aspect ratio wings or wings with very low sweep have been neglected.

Acknowledgment

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Similarity Rule for Sidewall Boundary-Layer Effect in Two-Dimensional Wind Tunnels

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Introduction

THE interference which the walls of a wind tunnel exert on the flow in the tunnel has long been a subject of concern and study. In the case of the two-dimensional or nearly two-dimensional wind tunnels which are used to test airfoils, the interference effects caused by the sidewalls are substantially different from those caused by the upper and lower walls.

The interference caused by the upper and lower walls of two-dimensional wind tunnels is primarily inviscid. The principal modification made to the upper and lower walls to relieve interference is ventilation with holes (pores) or longitudinal slots to relieve blockage effects. A summary of analytical methods which have been developed to predict blockage and lift interference effects in subsonic tunnels with closed, open, and ventilated upper and lower walls is given in Ref. 1.

The origin of sidewall interference in two-dimensional wind tunnels is viscous. The two sidewall interference problems which have received the most attention are the flat-plate-type growth of the sidewall boundary layer and the separation of the sidewall boundary layer due to large model-induced pressure gradients. Very little attention has been paid to the intermediate problem of the interaction of attached sidewall boundary layers with model-induced pressure gradients. Recently, experimental results were presented in Refs. 2 and 3 which can be used to evaluate this effect quantitatively. In the present paper, a simple analysis is presented which results in a similarity rule that relates compressibility and the interaction effect of the sidewall boundary layer to the model-induced pressure field. It is shown that this similarity rule is consistent with the relevant data in Refs. 2 and 3. An earlier version of the present study is given in Ref. 4.

Analysis

Consider steady, isentropic, small-perturbation flow in a nominally two-dimensional airfoil wind tunnel. Let the Cartesian coordinates in the freestream, normal, and spanwise directions be x, y, and z, and the respective velocity components be u, v, and w. The effective tunnel width is $b-2\delta^*$ where b and δ^* are the tunnel width and the sidewall displace thickness. It is assumed that δ^* can vary slightly with respect to x and y, and that the boundary conditions for the airfoil model and the upper and lower walls are independent of z. It is also assumed that the tunnel is narrow enough for flow at each sidewall to be strongly influenced by the other sidewall boundary layer. To lowest order, the spanwise velocity component in this tunnel varies linearly with the spanwise coordinate z as

$$w=-\frac{2uz}{b}\frac{\partial\delta^*}{\partial x} \tag{1}$$

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In wide tunnels, the disturbance caused by a sidewall boundary layer decays nonlinearly with distance from the sidewall so that Eq. (1) is not valid.

The flow in the wind tunnel described above is governed by the small perturbation form of the continuity equation, which can be written as

$$(1-M^2)\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=-\frac{\partial w}{\partial z}=\frac{2u}{b}\frac{\partial \delta^*}{\partial x} \quad (2)$$

where M is the Mach number. Because Eq. (2) and the boundary conditions do not depend on the coordinate z, the small perturbation solution for the velocity components u and v, the local Mach number M, and the thermodynamic properties will be independent of z.

The dynamics of the sidewall boundary layer are modeled with the von Kármán momentum integral, which can be written as

$$\frac{\partial \delta^*}{\partial x}=-\frac{\delta^*}{u}(2+H-M^2)\frac{\partial u}{\partial x}+\frac{\delta^*}{H}\frac{\partial H}{\partial x}+\frac{\tau_w}{\rho u^2} \quad (3)$$

where ρ is the density and δ^*, τ_w, and H are the sidewall displacement thickness, shearing stress, and shape factor. For the present problem, Eq. (3) can be simplified because the sidewall boundary layer in most airfoil wind tunnels can be approximated as a flat-plate boundary layer with a large Reynolds number and an equivalent length of the order of $\delta^*/(\tau_w/\rho u^2)$. In general, the model chord c is much smaller than the boundary-layer equivalent length at the model station so that the inequality

$$\frac{\tau_w}{\rho u^2} \ll \frac{\delta^*}{c} \quad (4)$$

applies, and, as a result, the last term in Eq. (3) can be neglected in the first approximation. As shown in Ref. 5, the shape factor for boundary layers with constant total temperature can be approximated as

$$H=(\bar{H}+1)\left\{1+\frac{\gamma-1}{2}M^2\right\}-1 \quad (5)$$

where $\bar{H}$ is the transformed shape factor. Because $\bar{H}$ approaches one as the Reynolds number becomes large, Eq. (5) can be written as

$$H=1+(\gamma-1)M^2 \quad (6)$$

for the present problem. From Eq. (6) and the small perturbation form of the energy equation, it follows that

$$\frac{\partial H}{\partial x}=\frac{(H-1)(H+1)}{u}\frac{\partial u}{\partial x} \quad (7)$$

With Eqs. (3) and (7) and inequality (4), Eq. (2) can be approximated as

$$\left\{1-M^2+\frac{2\delta^*}{b}\left[2+\frac{1}{H}-M^2\right]\right\}\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (8)$$

The terms in Eq. (8) which are due to the sidewall boundary layer are similar to the compressibility term and the divergence term in the x direction.

In the subsonic speed regime, Eq. (8) can be linearized because the local Mach number M can be approximated by the freestream value M_∞ and because constant approximate values for δ^* and H can be used. As a result, the pressure coefficients and integrals of the pressure coefficients for different subsonic flows can be related with the Prandtl-Glauert rule. For example, the nearly two-dimensional normal-force coefficient $\bar{C}_n$ in a wind tunnel with a sidewall boundary layer is related to the two-dimensional normal-force coefficient C_n at the same freestream Mach number M_∞ in the same tunnel with no sidewall boundary layer by the equation

$$\bar{\beta}\bar{C}_n=\beta C_n \quad (9)$$

where

$$\bar{\beta}=\sqrt{1-M_\infty^2+\frac{2\delta^*}{b}\left\{2+\frac{1}{H}-M_\infty^2\right\}} \quad (10)$$

$$\beta=\sqrt{1-M_\infty^2} \quad (11)$$

The boundary conditions for the upper and lower tunnel walls and the model boundary condition have not been used in the present derivation. Therefore, the present results for sidewall interference depend upon the details of the upper and lower walls and the model only in the way these quantities affect the variables M, H, and δ^* in Eq. (8). If approximate constant values are used for these quantities, the sidewall-interference predictions are independent of the nature of the upper and lower walls and the details of the model such as its size and shape. The upper and lower wall subsonic interference predictions given in Ref. 1 are influenced by the sidewall interference to the extent that these predictions are a function of $\bar{\beta}$ given by Eq. (10), rather than β given by Eq. (11).

Comparison with Experiment

The experiment described in Refs. 2 and 3 was performed in the ONERA R1Ch wind tunnel, which is sketched on the left side of Fig. 1. This is a high-pressure blowdown tunnel with a height and width of 38 and 8 cm, respectively, which can be fitted with either solid or porous upper and lower walls. There is a porous plate on the sidewall upstream of the model which is 50 cm long and which ends about 20 cm upstream of the model leading edge. Suction can be applied to this plate to remove mass from the sidewall boundary layer and hence control the sidewall boundary-layer thickness at the model.

The data of present interest are the measurements of the normal force on models at fixed angles of attack for different sidewall boundary-layer thicknesses. The sidewall boundary layer was measured near the model station in an empty tunnel for various values of the sidewall suction rate. Then the chordwise pressure distributions on the models were measured for the same sidewall suction rates. The normal-force coefficients were obtained from these pressure distributions.

A comparison of the experimental and theoretical results for the effect of the sidewall boundary-layer displacement thickness parameter $2\delta^*/b$ on the normal-force coefficient of a NACA 0012 airfoil at an angle of attack of 10 deg in the R1Ch wind tunnel with solid upper and lower walls is given on the right side of Fig. 1. In this paper, the shape factor is assumed to have the value 1.25.

Test results for models of the LC 100D supercritical airfoil with chords of 6 and 11 cm are also presented in Ref. 2, and

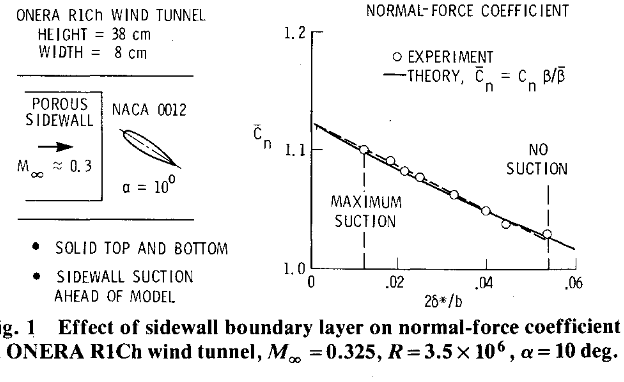

Fig. 1 Effect of sidewall boundary layer on normal-force coefficient in ONERA R1Ch wind tunnel, $M_\infty = 0.325$, $R = 3.5 \times 10^6$, $\alpha = 10$ deg.

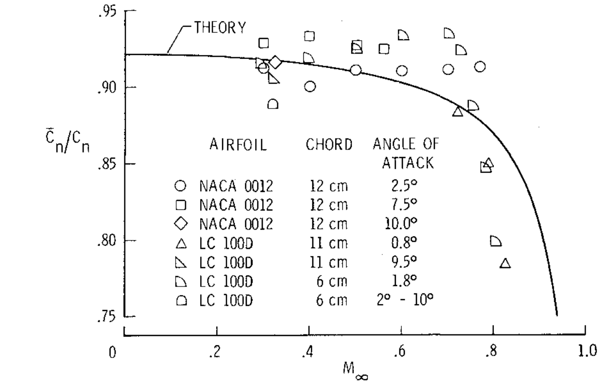

Fig. 2 Comparison of theory with data obtained in ONERA R1Ch wind tunnel with no sidewall suction upstream of model.

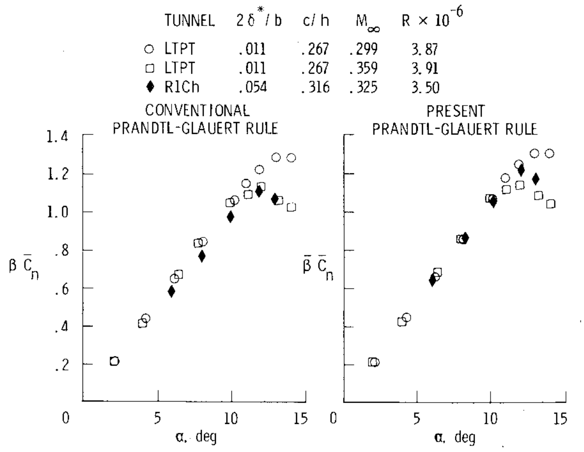

Fig. 3 Comparison of conventional and present form of Prandtl-Glauert rule for NACA 0012 airfoil.

additional data for the NACA 0012 airfoil and both models of the LC 100D airfoil are presented in Ref. 3. In Fig. 2, the experimental results for the normal-force coefficient ratio $\bar{C}_n/C_n$ obtained from Ref. 2 and a representative sample of the results obtained from Ref. 3 are compared with theoretical results obtained from Eq. (9). The subsonic results indicate good quantitative agreement as the incompressible limit is approached. The data for the supercritical airfoil show the same rapid decrease in $\bar{C}_n/C_n$ predicted by the theory as the freestream Mach number M_∞ increases toward one. The theoretical results for $\bar{C}_n/C_n$ for Mach numbers near the critical value differ from the experimental solution for both airfoils. This difference probably occurs because the theoretical solution for $\bar{C}_n/C_n$ does not account for nonlinear transonic effects.

For values of M_∞ greater than about 0.84 the experimental values for $\bar{C}_n/C_n$ for the supercritical LC 100D airfoil are greater than one. The experimental values for $\bar{C}_n/C_n$ for the NACA 0012 airfoil exceed one for values of M_∞ just larger than the critical value, and hence much smaller than the value for the supercritical airfoil. In fact, the effect occurs for the NACA 0012 airfoil before the beginning of the decrease in $\bar{C}_n/C_n$ caused by the singularity at $M_\infty = 1$. None of the experimental values for $\bar{C}_n/C_n$ greater than one are shown in Fig. 2. The effect is undoubtedly due to the presence of shock waves on the airfoils and the interaction for these shock waves with the sidewall boundary layers. The reason the LC 100D airfoil retains subsonic characteristics to higher freestream Mach numbers than the NACA 0012 airfoil is probably due to its supercritical design.

It is shown in Fig. 3 that the present theory can be used to correlate results for the variation of the normal-force coefficient with angle of attack obtained in wind tunnels with different values of the sidewall boundary-layer parameter. The airfoil used is the NACA 0012. The wind tunnels in which the data were obtained are the NASA Low-Turbulence Pressure Tunnel, which has closed walls, and the ONERA R1Ch tunnel with closed upper and lower walls. The displacement thickness parameter for the low-turbulence pressure tunnel is $2\delta^*/b = 0.011$. The data from this tunnel, which have not been published previously, were obtained by C. L. Ladson. The R1Ch data shown in Fig. 3 were obtained with no sidewall suction. It can be seen that the traditional function $\beta\bar{C}_n$ does not correlate the data between the two wind tunnels, but that the function $\bar{\beta}\bar{C}_n$ does.

The results presented in Fig. 3 are not corrected for interference from the closed upper and lower walls because the corrections for the two model-and-tunnel combinations used are almost the same. The analysis of Ref. 1 shows that the difference in the uncorrected normal-force coefficients for the two experiments due to interference from the upper and lower walls is only about 0.6% of the coefficient value.

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Improved Orthogonality Check for Measured Modes

80006

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Conventional Method

MANY structural dynamicists reply on an orthogonality check as a measure of the validity of normal modes derived from ground vibration testing. In order to perform this check, a mass model of the structure is used which has degrees of freedom that correspond to the points at which the modes are measured. This model is conventionally obtained by performing an approximate coordinate reduction, such as that of Guyan,[1] on a larger analytical model. Because this reduction should contain frequency dependent effects,[2,3] it is possible that it may give misleading results, especially when higher frequency modes are checked for orthogonality.

Improved Method

Consider the partitioned mass and stiffness matrices for a linear, undamped representation of a structure, where the upper left-hand coordinates represent the test points.

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